Quantization of an interacting spin-$\frac{3}{2}$ field and the Δ isobar

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Quantization of the free and interacting Rarita-Schwinger field is considered using the Hamiltonian path-integral formulation. The particular interaction we study in detail is the $\pi N\Delta$ coupling used in the phenomenology of the pion-nucleon and nucleon-nucleon systems. Within the Dirac constraint analysis, we show that there is an excess of degrees of freedom in the model, as well as the inconsistency related to the Johnson-Sudarshan-Velo-Zwanzinger problem. It is further suggested that couplings invariant under the gauge transformation of the Rarita-Schwinger field are generally free from these inconsistencies. We then construct and briefly analyze some lowest in derivatives gauge-invariant $\pi N\Delta$ couplings. [S0556-2821(98)01919-5]

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I. INTRODUCTION

A covariant description of the interacting spin-3/2 field is famous for its various problems and paradoxes. Presently, supergravity is the only example of a local field theory which includes a massless spin-3/2 field (gravitino) in a consistent way, for a review see Ref. [1]. For the particle phenomenology, however, it would be desirable to construct a consistent description in a flat space. Such a description is needed, for example, for the treatment of the spin-3/2 baryon resonances, such as the $\Delta(1232)$ isobar, in low-energy hadron scattering [2–7]. Another interesting application is the search for the spin-3/2 leptons [8].

The major problems in the local[1] higher-spin field theory are closely related to the presence of unphysical lower-spin components in the covariant representation of the field. More specifically, a field with a given spin $s \geqslant 1$, in addition to the physical components, necessarily contains components of spin $(s-1)$, $(s-2)$, etc. For instance, in the Rarita-Schwinger (RS) formalism [10] adopted in this work, the spin-3/2 field is represented by a 16 component vector-spinor ψ_μ, while only 4 components are needed for the description of a massive spin-3/2 particle and thus the rest of the components should be attributed to the lower-spin sector. The free action of such theories is then constructed in such a way that at the level of the equations of motions the constraints are produced reducing the number of independent components to the necessary value (equal to $2s+1$ for a massive and 2 for a massless particle with spin).

In the interacting case the situation is generally more complex, since all the components may couple in a nontrivial way. The constraints are then altered, moreover their amount may change. In the latter case, i.e., if the number of constraints in the free and interacting theory is different, one can conclude that a wrong number of degrees of freedom (DOF) is interacting, and therefore, this form of interaction is physically unacceptable. Another type of inconsistency which may often arise is the presence of the famous Johnson-Sudarshan (JS) [11], and Velo-Zwanzinger (VZ) [12] problems. More recently, it was shown that JS and VZ problems have a common origin [13], and furthermore they are related to the mentioned problem of the constraint violation [14,15].

All these problems are known [2,16,17] to be present for the coupling of a massive RS field $\psi_\mu(x)$ to a spinor $\Psi(x)$ and a (pseudo-) scalar $\phi(x)$ described by the following Lagrangian:[2]

$$\mathcal{L}_{\rm int} = g\,\overline{\psi}_\mu(g^{\mu\nu} + a\,\gamma^\mu\gamma^\nu)\Psi\,\partial_\nu\phi + \text{H.c.}, \tag{1}$$

where g is the coupling constant, and a is related to the *off-shell parameter z* as follows: $a=-z-\frac{1}{2}$, cf. Ref. [2]. Up to the isospin complications, this interaction represents the $\pi N\Delta$ coupling, frequently used in various field-theoretical models of the low-energy πN and NN interactions.[3]

This coupling is also known to have the above-mentioned bad property of involving the unphysical spin-1/2 components. The contribution of the spin-1/2 sector exhibits itself in the Δ-exchange scattering amplitudes as a substantial spin-1/2 background in addition to the spin-3/2 resonance behavior around the Δ mass position.

In the present work, the pathologies of this coupling are analyzed within the Dirac-Faddeev (DF) quantization framework [18–24]. Thus, first we shall transit to the Hamiltonian formulation, find the constraints in the phase-space of the theory using the Dirac's method [18] and check whether the abovementioned DOF counting is consistent. Secondly, we shall write down the phase-space path integral taking the constraints into account, following a generalization [20] of Faddeev's approach [19]. It is usually possible to integrate out the conjugate momenta and thus obtain the configuration-space path integral. The obtained path integral can in principle be different from the one we would naively write down without taking the constraints into account. In

[1]For nonlocal formulations see [9].

[2]The conventions used throughout this paper are: $\hbar=c=1$, $g_{\mu\nu}=\text{diag}(1,-1,-1,-1)$, $\varepsilon^{0123}=1$, $\gamma_5=i\gamma_0\gamma_1\gamma_2\gamma_3$, $\sigma_{\mu\nu}=\frac{1}{2}[\gamma_\mu,\gamma_\nu]$, spinor indices are usually omitted.

[3]For some applications to the πN system see, e.g., Refs. [2,3] (effective chiral Lagrangians), [6] (relativistic meson-exchange models), [7] (chiral perturbation theory), see also Ref. [4] for a list of common problems in the treatment of the Δ.

this case the naive Feynman rules (which one would just ''read off'' the original Lagrangian) are generally not applicable. Applying this procedure to interaction (1), indeed leads to a result different from the naive one, see Eq. (30). On the way to this result, we shall meet the inconsistencies at the classical level found before using different methods [2,16,17].

The question arises whether it is possible in principle to formulate a consistent interaction of the RS field without supersymmetry, or coupling to gravity, or both. As will be argued in Sec. IV, it is generally possible, if the interaction in question is symmetric under the gauge transformation of the RS field. In particular, we construct the following gauge-invariant $\pi N\Delta$ coupling

$$\mathcal{L}^{(\mathrm{GI})}_{\mathrm{int}} = g\varepsilon^{\mu\nu\alpha\beta}(\partial_\mu\overline{\psi}_\nu)\gamma_5\gamma_\alpha\Psi\partial_\beta\phi + \mathrm{H.c.},$$

which is shown to admit consistent path-integral quantization. The good properties of this interaction are especially clearly seen from the tree-level Δ-particle exchange amplitude, Eq. (49). It is proportional to the spin-3/2 projection operator, and, at the same time, is not singular at $p^2=0$. Consequently, the decoupling of the spin-1/2 component of the RS field is achieved in the manifestly covariant and local formulation. The spin-1/2 background is absent.

The paper is organized as follows. In the next section we work out the DF procedure for the free massive spin-3/2 field. This discussion serves mainly as an introduction to the formalism. In Sec. III we perform the Dirac constraint analysis of the conventional $\pi N\Delta$ interaction (1), notify the presence of the JS-VZ problem, and obtain the configuration-space path integral of the model. In Sec. IV we argue that gauge-invariant interactions do not, in general, alter the number of constraints, and consider some lowest in derivatives gauge-invariant $\pi N\Delta$ couplings. The conclusions are formulated in Sec. V. Finally, an extension of the Stückelberg formalism to the case of the spin-3/2 field is given in the Appendix.

II. FREE RARITA-SCHWINGER FIELD

The quantization of the free RS field in Hamiltonian formulation was considered previously in Refs. [25–28]. In this section we shall briefly recapitulate these considerations in order to summarize the results and set up the framework. Also, the free-field quantization is usually done on Majorana (Hermitian) field, while here we work with the complex field, hence allowing for the charge. This leads only to minor modifications related to the doubling of the field components and corresponding DOF and constraints.

The free Lagrangian of a complex RS field $\psi_\mu(x)$ with mass m is written as follows:

$$\begin{aligned}\mathcal{L}_{3/2} &= \tfrac{1}{2}\overline{\psi}_\mu\{\sigma^{\mu\nu},(i\partial\!\!\!/-m)\}\psi_\nu \\ &= -\tfrac{1}{2}\varepsilon^{\mu\nu\alpha\beta}\overline{\psi}_\mu\gamma_5\gamma_\alpha\partial_\beta\psi_\nu \\ &\quad + \tfrac{1}{2}\varepsilon^{\mu\nu\alpha\beta}(\partial_\beta\overline{\psi}_\mu)\gamma_5\gamma_\alpha\psi_\nu \\ &\quad - m\overline{\psi}_\mu\sigma^{\mu\nu}\psi_\nu . \end{aligned} \tag{2}$$

To determine the constraints we follow the path of Dirac [18]. From the definition of conjugate momenta,

$$\pi^{\mu\dagger}(x) = \partial\mathcal{L}/\partial\dot{\psi}_\mu(x), \quad \pi^\mu(x) = \partial\mathcal{L}/\partial\dot{\psi}^\dagger_\mu(x),$$

we find the following primary constraints:

$$\begin{aligned}&\theta_0(x)=\pi_0(x), \quad \theta_i(x)=\pi_i(x)+\tfrac{1}{2}\varepsilon_{ijk}\gamma_0\gamma_5\gamma_k\psi_j(x), \\ &\theta^\dagger_0(x)=\pi^\dagger_0(x), \quad \theta^\dagger_i(x)=\pi^\dagger_i(x)+\tfrac{1}{2}\varepsilon_{ijk}\psi^\dagger_j(x)\ \gamma_0\gamma_5\gamma_k .\end{aligned} \tag{3}$$

The Hamiltonian, $H=\int d^3x\,\mathcal{H}(x)$, is then given by

$$\begin{aligned}\mathcal{H}_{3/2} &= [\overline{\psi}_i(\varepsilon_{ijk}\gamma_5\gamma_j\partial_k - m\overline{\psi}_i\gamma_i\gamma_0)\psi_0 + \mathrm{H.c.}] \\ &\quad + \overline{\psi}_i(\varepsilon_{ijk}\gamma_0\gamma_5\partial_k + m\sigma_{ij})\psi_j .\end{aligned} \tag{4}$$

We also introduce the fundamental Poisson brackets (defined at $x_0=y_0$)

$$\{\psi_{\mu\sigma}(x),\,\pi^{\nu\dagger}_\tau(y)\}_P = \delta^\nu_\mu\delta_{\sigma\tau}\,\delta^3(x-y), \tag{5}$$

here we have written out the spinor indices $\sigma,\tau=0,\ldots,3$. In the following we will omit them again. Brackets involving only fields or only momenta vanish.[4]

The primary constraints should now be added to the Hamiltonian through the Lagrange multipliers to form the *total Hamiltonian* density

$$\mathcal{H}_T = \mathcal{H}_{3/2} + \lambda_0\theta_0 + \lambda_i\theta_i + \mathrm{H.c.} \tag{6}$$

To guarantee the conservation of constraints in time one requires that they commute with the total Hamiltonian, i.e., the corresponding Poisson bracket must vanish.

From condition $\{\theta_i(x),H_T\}_P=0$, the Lagrange multipliers λ_i can be determined. Constraints θ_i are thus second class and we may resolve them right away by introducing the Dirac bracket

$$\begin{aligned}\{A(x),B(y)\}_D &= \{A(x),B(y)\}_P \\ &\quad - \int d^3z_1\,d^3z_2\,\{A(x),\theta^\dagger_i(z_1)\}_P \\ &\quad \times(\{\theta^\dagger_i(z_1),\theta_j(z_2)\}_P)^{-1}\{\theta_j(z_2),B(y)\}_P .\end{aligned} \tag{7}$$

To this end we can find

$$\{\theta_i(x),\theta^\dagger_j(y)\}_P = -i\sigma_{ij}\delta^3(x-y), \tag{8}$$

$$(\{\theta_i(x),\theta^\dagger_j(y)\}_P)^{-1} = -\tfrac{1}{2}i\gamma_j\gamma_i\delta^3(x-y), \tag{9}$$

hence[5]

$$\{\psi_i(x),\psi^\dagger_j(y)\}_D = \tfrac{1}{2}i\gamma_j\gamma_i\delta^3(x-y). \tag{10}$$

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[4]From the property of the Poisson bracket, $\{A,B\}^\dagger_P = -\{B^\dagger,A^\dagger\}_P$, we have $\{\psi^\dagger_\mu(x),\,\pi^\nu(y)\}_P = -\delta^\nu_\mu\,\delta^3(x-y)$.

[5]One can get to this and some other results in a more efficient way by using the Hamiltonian reduction [29] instead of Dirac's analysis. (We thank L. D. Faddeev for this remark.)

From the condition that θ_0 and $\theta_0^\dagger$ commute with the (total) Hamiltonian we find the secondary constraints,[6]

$$\theta_4(x) = -i\sigma_{ij}\partial_i\psi_j + m\gamma_i\psi_i,$$
$$\theta_4^\dagger(x) = -i\partial_i\psi_j^\dagger\sigma_{ij} - m\psi_i^\dagger\gamma_i. \tag{11}$$

We may rewrite the Hamiltonian in the following fashion:

$$\mathcal{H}_{3/2} = \theta_4^\dagger\psi_0 + \psi_0^\dagger\theta_4 + \overline{\psi_i}(\varepsilon_{ijk}\gamma_0\gamma_5\partial_k + m\sigma_{ij})\psi_j. \tag{12}$$

Now one can immediately see that the tertial constraints θ_5 (and $\theta_5^\dagger$) arising from $\{\theta_4(x), H_T\}_D = 0$ [and $\{\theta_4^\dagger(x), H_T\}_D = 0$] are linear in ψ_0 ($\psi_0^\dagger$) with the following proportionality coefficient:

$$\int d^3x\{\theta_4(x), \theta_4^\dagger(y)\}_D = \tfrac{3}{2}im^2. \tag{13}$$

Clearly, the conditions that θ_5 and $\theta_5^\dagger$ commute with the total Hamiltonian determine the remaining Lagrange multipliers λ_0 and $\lambda_0^\dagger$, thus no more constraints arise. It is also clear that all the constraints are second class.

We can perform now an exercise in the DOF counting. The field ψ_μ and its conjugate momentum π^μ have $4\times4 = 16$ (complex) components each, so 32 in total. We have $6\times4 = 24$ (complex) constraints on them. Hence the number of independent components is 8: precisely what is needed for the description of the spin DOF in the phase-space of a massive spin-3/2 particle.

In the massless case the situation is somewhat different. The requirement

$$\{\theta_4(x), H_T\}_D = 0$$

becomes an identity, and no θ_5 constraints arise. We then have only five fermionic constraints, where θ_i are second class while θ_0 and θ_4 are first class. The appearance of the first-class constraints is, of course, related to the fact that the massless Lagrangian is (up to a total derivative) invariant under the gauge transformation,

$$\psi_\mu \to \psi_\mu + \partial_\mu\epsilon, \tag{14}$$

where $\epsilon(x)$ is a complex fermionic field. To each first-class constraint we have to introduce a gauge-fixing condition. The DOF counting is then also consistent: we are left with four independent field components in the phase-space which is appropriate for a massless particle with spin.

Let us now proceed to the path-integral quantization of the system. We concentrate on the massive case. Following the generalization of Faddeev's procedure [19] to the case of (fermionic) second-class constraints [20–23] we write down the phase-space path integral in the following form:

$$Z = \int \mathcal{D}\psi_\mu\,\mathcal{D}\psi_\mu^\dagger\,\mathcal{D}\pi^\mu\,\mathcal{D}\pi^{\mu\dagger}\,(\det\|\{\theta,\theta\}_P\|)^{1/2} \times\prod_{n=0}^{5}\delta(\theta_n)\,\delta(\theta_n^\dagger) \times\exp\left\{i\int d^4x[\pi^{\mu\dagger}\dot\psi_\mu + \dot\psi_\mu^\dagger\pi^\mu - \mathcal{H}_{3/2}]\right\}, \tag{15}$$

where $\|\{\theta,\theta\}_P\|$ represents the matrix of Poisson brackets of constraints. In our case it is[7]

$$\begin{pmatrix} 0 & \|\{\theta(x),\theta^\dagger(y)\}_P\| \\ \|\{\theta^\dagger(x),\theta(y)\}_P\| & 0 \end{pmatrix} \tag{16}$$

where

$$\|\{\theta(x),\theta^\dagger(y)\}_P\| = \begin{pmatrix} 0 & 0 & 0 & \frac{3}{2}im^2 \\ 0 & -i\sigma_{ij} & \Gamma_i & \{\theta_i,\theta_5^\dagger\} \\ 0 & \Gamma_j & 0 & \{\theta_4,\theta_5^\dagger\} \\ \frac{3}{2}im^2 & \{\theta_5,\theta_j^\dagger\} & \{\theta_5,\theta_4^\dagger\} & \{\theta_5,\theta_5^\dagger\} \end{pmatrix}\delta^3(x-y), \qquad \Gamma_i \equiv i\sigma_{ij}\partial_j + m\gamma_i. \tag{17}$$

The calculation of the determinant and integration over π's produce the following result:

$$Z = \int \mathcal{D}\psi_\mu\,\mathcal{D}\psi_\mu^\dagger\det[(i\gamma_i\partial_i + \tfrac{3}{4}m)\,\delta^3(x-y)] \times\exp\left(i\int\mathcal{L}_{3/2}\right). \tag{18}$$

The determinant is field independent and can be dropped, we have kept it just for further comparison to the interacting case. Having obtained path integral (18) we complete the DF quantization of the free massive spin-3/2 field and conclude that constraints do not modify the original Lagrangian, hence the ''naive'' Feynman rules apply.

We will not treat separately the massless case (this is done in details in Refs. [25,26]). Instead, we may apply an analog of the Stückelberg mechanism [31], which allows us to treat the massless and massive case on the same footing. This analysis is done in the Appendix.

III. THE $\pi N\Delta$-COUPLING MODEL

In this section we apply the Dirac-Faddeev procedure to quantize the $\pi N\Delta$ phenomenological interaction discussed

[6]Note the identities: $\varepsilon_{ijk}\gamma_5\gamma_k = -i\sigma_{ij}\gamma_0$, $\frac{1}{2}i\varepsilon_{ijk}\gamma_j\gamma_k = \gamma_5\gamma_0\gamma_i$, $\frac{1}{2}\varepsilon_{ijk}\varepsilon_{lmn}\gamma_j\gamma_m\gamma_k\gamma_n = -\sigma_{il}$.

[7]Note that nowhere in our calculation do we need to know the θ_5 constraint fully. It suffices to know that θ_5 is linear in ψ_0 with the already determined coefficient $\frac{3}{2}im^2$. This observation has been made also in Ref. [30].

in the Introduction. The model is given by the following Lagrangian:

$$\begin{aligned}
&\mathcal{L}=\mathcal{L}_0+\mathcal{L}_{1/2}+\mathcal{L}_{3/2}+\mathcal{L}_{\rm int},\\
&\mathcal{L}_0=\tfrac{1}{2}\partial_\mu\phi\,\partial^\mu\phi-\tfrac{1}{2}\mu^2\phi^2,\\
&\mathcal{L}_{1/2}=\overline{\Psi}(i\not\partial-M)\Psi\\
&\quad=\tfrac{1}{2}i\overline{\Psi}\gamma_\mu\partial_\mu\Psi-\tfrac{1}{2}i(\partial_\mu\overline{\Psi})\gamma_\mu\Psi-M\overline{\Psi}\Psi,
\end{aligned}\tag{19}$$

where $\mathcal{L}_{\rm int}$ and $\mathcal{L}_{3/2}$ are defined in Eqs. (1) and (2), respectively.

We follow precisely the same steps as in the preceding section. In addition to π^μ we define

$$\begin{aligned}
&P(x)=\partial\mathcal{L}/\partial\dot\phi(x),\\
&\Pi^\dagger(x)=\partial\mathcal{L}/\partial\dot\Psi(x),\quad \Pi(x)=\partial\mathcal{L}/\partial\dot\Psi^\dagger(x),
\end{aligned}\tag{20}$$

and find the ''velocity'' $\dot\phi$,

$$\begin{aligned}
&\dot\phi(x)=P(x)-F[\Psi(x),\psi_\mu(x)],\\
&F[\Psi,\psi_\mu]\equiv g(1+a)\,\overline{\psi_0}\Psi-ga\,\overline{\psi_i}\gamma_i\gamma_0\Psi+\text{H.c.},
\end{aligned}\tag{21}$$

and the following primary constraints [in addition to Eq. (3)]:

$$\chi(x)=\Pi(x)-\tfrac{1}{2}i\Psi(x),\quad \chi^\dagger(x)=\Pi^\dagger(x)+\tfrac{1}{2}i\Psi^\dagger(x).\tag{22}$$

The model Hamiltonian is given by

$$\begin{aligned}
&\mathcal{H}=\mathcal{H}_0+\mathcal{H}_{1/2}+\mathcal{H}_{3/2}+\mathcal{H}_{\rm int},\\
&\mathcal{H}_0=\tfrac{1}{2}(P^2-F^2)+\tfrac{1}{2}(\partial_i\phi)^2+\tfrac{1}{2}\mu^2\phi^2,\\
&\mathcal{H}_{1/2}=\overline{\Psi}(i\gamma_i\partial_i+M)\Psi,\\
&\mathcal{H}_{\rm int}=-\mathcal{L}_{\rm int}=-(P-F)\,F+g[a\,\overline{\psi_0}\gamma_0\gamma_i\partial_i\phi\\
&\qquad+\overline{\psi_i}(\delta_{ij}-a\,\gamma_i\gamma_j)\partial_j\phi]\Psi+\text{H.c.}
\end{aligned}\tag{23}$$

with $\mathcal{H}_{3/2}$ given in Eq. (4).

We postulate the fundamental Poisson brackets

$$\begin{aligned}
&\{\phi(x),P(y)\}_P=\delta^3(x-y),\\
&\{\Psi_\sigma(x),\Pi^\dagger_\tau(y)\}_P=\delta_{\sigma\tau}\,\delta^3(x-y),\\
&\{\psi_{\mu\sigma}(x),\,\pi^{\nu\dagger}_\tau(y)\}_P=\delta^\nu_\mu\,\delta_{\sigma\tau}\,\delta^3(x-y);
\end{aligned}\tag{24}$$

all the other brackets vanish. Note that the brackets are symmetric in the case of fermionic variables (such as $\Psi,\Pi,\psi_\mu,\pi^\mu$) and antisymmetric in the case of bosonic variables (such as $\phi,P,\mathcal{H}$), they are also antisymmetric in the mixed case.

Next we resolve the second-class constraints θ_i and χ by introducing corresponding Dirac brackets, and note

$$\{\Psi(x),\Psi^\dagger(y)\}_D=-i\,\delta^3(x-y)\tag{25}$$

while $\{\psi_i(x),\psi^\dagger_j(y)\}_D$ is given by Eq. (10).

A crucial point here is that the condition of conservation of θ_0 constraint leads to a constraint which in general contains ψ_0. Namely,

$$\begin{aligned}
\theta_4=&-i\sigma_{ij}\partial_i\psi_j+m\gamma_i\psi_i-ag\gamma_i\Psi\partial_i\phi\\
&+g(1+a)\,(P-F)\,\gamma_0\Psi
\end{aligned}\tag{26}$$

and similar for $\theta_4^\dagger$. It is F that has an explicit dependence on ψ_0 as given by Eq. (21).

As we saw in the previous section, the constraint containing ψ_0 is always the last one in the chain of constraints. Hence for $a\neq-1$, θ_4 is the last constraint, and we have then 5 ($\times4$) constraints, all of them being second class. Counting the number of DOF for this case we certainly find an excess of them, because we are one constraint too short as compare to the free case where the DOF counting is built in correctly. Thus, we conclude that for $a\neq-1$ the $\pi N\Delta$ interaction considered here is *inconsistent* with the free theory construction. The same conclusion has been drawn by Nath, Etemadi, and Kimel [2] based on a constraint analysis in Lagrangian formulation. The choice $a=-1$ is thus preferable and we continue the analysis for this case only.

For $a=-1$, the θ_4 constraints read

$$\begin{aligned}
&\theta_4(x)=-i\sigma_{ij}\partial_i\psi_j+m\gamma_i\psi_i+g\gamma_i\Psi\partial_i\phi,\\
&\theta_4^\dagger(x)=-i\partial_i\psi_j^\dagger\sigma_{ij}-m\psi_i^\dagger\gamma_i-g\Psi^\dagger\gamma_i\partial_i\phi.
\end{aligned}\tag{27}$$

As in the free case, constraints θ_5 and $\theta_5^\dagger$ are linearly proportional to ψ_0. Now only with a different coefficient

$$R(x)\equiv\int d^3y\,\{\theta_4(x),\theta_4^\dagger(y)\}_D=i[\tfrac{3}{2}m^2-g^2(\partial_i\phi)^2].\tag{28}$$

At this point we hit another problem. The coefficient may vanish when $\frac{3}{2}m^2=g^2(\partial_i\phi)^2$. Then, either the θ_4 constraints are first class, or we will find some further second-class constraints. In any case the DOF counting will again be different from that of the free theory. In the massless case the situation is even worse since the problem occurs for any value of $g^2(\partial_i\phi)^2$.

It is interesting to note that the same problem arises in the constraint analysis of the minimal coupling of the RS field to the external electromagnetic field [14,15]. There it was identified with the JS-VZ problem. On the other hand, Hagen [16] and Singh [17] revealed the JS and VZ problems in the $\pi N\Delta$ coupling being considered. Their analysis is done in lines with the original treatment [11,12] and thus is rather different from ours; nevertheless, the factor giving rise to the JS and VZ problem in their works is precisely $R(x)$ of Eq. (28). Moreover, we can easily compute the field commutators taking into account θ_4 constraints (i.e., the second stage Dirac bracket), and find that the corresponding quantum commutators are not positive definite, because R is not, in line with Hagen's conclusion. We can therefore confirm the

observation of [13,14] that the JS-VZ problem appears itself in the violation of constraints.

To proceed with the quantization let us assume $R(x)\neq 0$ (although note that this is not a Lorentz-invariant condition), and write down the path integral. According to Eq. (15) we need

$$(\det\|\{\theta,\theta\}_P\|)^{1/2}=\det\begin{pmatrix} 0 & 0 & 0 & R & 0 \\ 0 & -i\sigma_{ij} & \Gamma_i & \{\theta_i,\theta_5^\dagger\} & 0 \\ 0 & \Gamma_j & 0 & \{\theta_4,\theta_5^\dagger\} & g\gamma_i\partial_i\phi \\ R & \{\theta_5,\theta_j^\dagger\} & \{\theta_5,\theta_4^\dagger\} & \{\theta_5,\theta_5^\dagger\} & \{\theta_5,\chi^\dagger\} \\ 0 & 0 & g\gamma_i\partial_i\phi & \{\chi,\theta_5^\dagger\} & -i \end{pmatrix}\delta^3(x-y). \tag{29}$$

Simplifying this determinant and carrying out the integration over the conjugate momenta we obtain

$$Z=\int \mathcal{D}\psi_\mu\,\mathcal{D}\psi_\mu^\dagger\,\mathcal{D}\Psi\,\mathcal{D}\Psi^\dagger\,\mathcal{D}\phi \times\det\left[\left(i\gamma_i\partial_i+\tfrac{3}{4}m-\frac{g^2}{2m}(\partial_i\phi)^2\right)\delta^3(x-y)\right] \times\exp\left(i\int\mathcal{L}\right). \tag{30}$$

Thus, our final path integral differs from the naive path integral by the nontrivial determinant entering the measure.

Noncovariant field-dependent determinants do often occur in the Hamiltonian path-integral quantization of systems with second-class constraints, see Refs. [33,34]. Usually their contributions to the Green functions is canceled by the singular terms coming from the time-ordering operators, so that resulting Green functions are covariant. It would be interesting to see whether this mechanism occurs also in the case of Eq. (30) or, perhaps, there is indeed some breaking of Lorentz symmetry suggested by the presence of the JS-VZ problem.

IV. GAUGE-INVARIANT COUPLINGS

In the previous section we have seen that the conventional $\pi N\Delta$ interaction suffers from inconsistencies related to the violation of constraints, in particular the JS-VZ problem. On the other hand, it is intuitively clear that (i) gauge-invariant couplings are generally consistent with the DOF counting. Indeed, the number of constraints is related to the number of local symmetries of the Lagrangian, while gauge-invariant couplings do not destroy the symmetry of the free RS Lagrangian where the DOF counting is correct. We can prove statement (i) more rigorously for the linear couplings of the RS field, i.e., the case when the interaction Lagrangian is given by

$$\mathcal{L}_{\rm linear}=\psi_\mu^\dagger J^\mu+\text{H.c.}, \tag{31}$$

J^μ is independent of ψ_μ The proof proceeds as follows [we basically follow the proof of Eq. (8.2.5) in Ref. [32]].

If other fields do not change under the gauge transformation, we can concentrate just on the ψ_μ dependent part of the Lagrangian, which is

$$\mathcal{L}=\mathcal{L}_{3/2}+\mathcal{L}_{\rm linear}.$$

The gauge invariance of the massless Lagrangian then implies

$$\partial_\mu J^\mu=0. \tag{32}$$

Determining the constraints, we find the usual primary constraints

$$\theta_0=\frac{\partial\mathcal{L}}{\partial(\partial_0\psi_0)} \tag{33}$$

and θ_i of Eq. (3). The θ_i constraints do not produce any secondary constraints, while requiring time independence of θ_0 gives us the usual

$$\theta_4=\partial_0\theta_0. \tag{34}$$

Now, using the Euler-Lagrange field equations and Eq. (32), we obtain

$$\theta_5\equiv\partial_0\theta_4=m\sigma^{\mu\nu}\partial_\mu\psi_\nu. \tag{35}$$

Thus, only the mass term can affect the number of constraints, which proves (i) for the case of linear coupling.

According to (i) it seems promising to search for consistent $\pi N\Delta$ couplings among the gauge-invariant ones. The simplest way to construct those is to couple the RS field to an explicitly conserved current. (Actually, the only other way we can see is to allow the pion and the nucleon field also transform under the gauge transformation, similarly to how they transform under the photon gauge transformation. This, however, would obviously require a supersymmetric realization. Although an interesting possibility, here we restrict ourselves to nonsupersymmetric realizations.)

The lowest in derivatives explicitly gauge-invariant $\pi N\Delta$ interaction is given by the following Lagrangian:

$$\mathcal{L}_{\rm int}=g(\partial_\mu\overline{\psi}_\nu)\sigma^{\mu\nu}\Psi\phi+\text{H.c.} \tag{36}$$

However, this interaction is in some sense trivial: it describes the coupling of the nucleon and pion to $\partial\cdot\psi$ and $\gamma\cdot\psi$, i.e., the spin-1/2 sector of the Δ field. Furthermore, the corresponding tree-level Feynman amplitude for the πN scattering through a virtual Δ exchange

$$M(p)=\Gamma^{\alpha}(p)S_{\alpha\beta}(p)\Gamma^{\beta}(p), \tag{37}$$

where p is four-momentum of the Δ, $\Gamma^{\alpha}(p)$ and $S_{\alpha\beta}(p)$ are the naive Feynman rules for the vertex and the RS propagator, respectively,

$$\Gamma^{\alpha}(p)=g\sigma^{\alpha\mu}p_{\mu}, \tag{38}$$

$$S_{\alpha\beta}(p)=\frac{\not p+m}{p^2-m^2}\left[g_{\alpha\beta}-\tfrac{1}{3}\gamma_{\alpha}\gamma_{\beta}-\frac{1}{3m^2}(\not p\gamma_{\alpha}p_{\beta}+p_{\alpha}\gamma_{\beta}\not p)\right], \tag{39}$$

vanishes exactly: $M(p)=0$, for all p. Having such a classically ''invisible'' Δ is maybe interesting in some scenarios, but certainly not in the applications we are interested in here. We thus should conclude that the $\pi N\Delta$ interaction (36) involves a correct number of Δ's field components, however, they have wrong spin representing parts of the spin-1/2 sector of the RS field, consequently this interaction can not describe a physical coupling to the spin-3/2 particle.

The next lowest in derivatives gauge-invariant interaction is written down in the Introduction, and reads as follows:

$$\mathcal{L}_{\rm int}=g\varepsilon^{\mu\nu\alpha\beta}(\partial_{\mu}\overline{\psi}_{\nu})\gamma_5\gamma_{\alpha}\Psi\partial_{\beta}\phi+\text{H.c.} \tag{40}$$

For this interaction the tree-level amplitude does not vanish. Moreover, the result is not sensitive to $1/m^2$ term of the RS propagator, thus a well-defined massless limit is guaranteed. We shall discuss the tree-level calculation in more detail, but first let us perform the DF quantization of this interaction.

To treat the massive and massless case simultaneously we introduce the Stückelberg spinor $\xi(x)$ described in the Appendix. Our model Lagrangian is thus defined by Eqs. (19), (A1), and (40).

The model has the following primary and secondary constraints (the Hermitian conjugates are omitted):

$$\begin{aligned}
\theta_i&=\pi_i-i\sigma_{ij}(\tfrac{1}{2}\psi_j+g\Psi\partial_j\phi),\\
\theta_S&=\eta-m\gamma_i\psi_i,\\
\chi&=\Pi-\tfrac{1}{2}i\Psi,\\
\theta_0&=\pi_0,\\
\theta_4&=-i\sigma_{ij}\partial_i\psi_j+m\gamma_i\psi_i+m\gamma_i\partial_i\xi-ig\sigma_{ij}\partial_i\Psi\partial_j\phi,
\end{aligned} \tag{41}$$

and the Hamiltonian density given by

$$\begin{aligned}
\mathcal{H}=&\tfrac{1}{2}(P-F)^2+\tfrac{1}{2}(\partial_i\phi)^2+\tfrac{1}{2}(\mu\phi)^2+\overline{\Psi}(i\gamma_i\partial_i+M)\Psi\\
&+[\psi_0^{\dagger}\theta_4+\tfrac{1}{2}\overline{\psi}_i(\varepsilon_{ijk}\gamma_0\gamma_5\partial_k+m\sigma_{ij})\psi_j+m\overline{\psi}_i\sigma_{ij}\partial_j\xi\\
&-g\varepsilon_{ijk}(\partial_i\psi_j^{\dagger})\gamma_5\Psi\partial_k\phi+\text{H.c.}],
\end{aligned} \tag{42}$$

where $F=-ig(\partial_i\psi_j^{\dagger})\sigma_{ij}\Psi+\text{H.c.}$, and θ_4 is given in Eq. (41). Once again, we introduce the Dirac bracket with respect to the second-class constraints (θ_i, θ_S, χ) and find that the field commutators remain to be given by Eqs. (A8), (25).

Now, as can be shown by a direct computation, but also follows from the proof given in the beginning of this section, the secondary constraint, θ_4, commutes with the total Hamiltonian. Hence, constraints (41) are all constraints in the model.

The construction of the path integral goes in exactly the same way as discussed in the Appendix. Although in this case the matrix of the second-class constraint Poisson brackets

$$\|\{\theta^{(2)},\theta^{\dagger(2)}\}_P\|=-\begin{pmatrix} i\sigma_{ij} & m\gamma_i & ig\sigma_{ik}\partial_k\phi\\ m\gamma_j & 0 & 0\\ ig\sigma_{jk}\partial_k\phi & 0 & i\end{pmatrix}\times\delta^3(x-y), \tag{43}$$

is field dependent, its determinant is not,

$$\det\|\{\theta^{(2)},\theta^{\dagger(2)}\}_P\|=\det[3im^2\delta^3(x-y)], \tag{44}$$

and can be neglected.

Taking the Coulomb gauge, integrating out the momenta and covarianizing the measure we obtain the following configuration-space path integral of the model:

$$Z=\int\mathcal{D}\psi_{\mu}\,\mathcal{D}\psi_{\mu}^{\dagger}\,\mathcal{D}\Psi\,\mathcal{D}\Psi^{\dagger}\,\mathcal{D}\phi\,\mathcal{D}\xi\,\mathcal{D}\xi^{\dagger}\delta(\gamma\cdot\psi)\times\delta(\psi^{\dagger}\cdot\gamma)\exp\left(i\int\mathcal{L}\right). \tag{45}$$

Another important simplification which occurs here due to the gauge symmetry is the decoupling of the Stückelberg spinor. We thus may easily integrate it out as well, obtaining

$$Z=\int\mathcal{D}\psi_{\mu}\,\mathcal{D}\psi_{\mu}^{\dagger}\,\mathcal{D}\Psi\,\mathcal{D}\Psi^{\dagger}\,\mathcal{D}\phi\,\delta(\gamma\cdot\psi)\,\delta(\psi^{\dagger}\cdot\gamma)\times\delta(\partial\cdot\psi)\,\delta(\partial\cdot\psi^{\dagger})\exp\left(i\int\mathcal{L}\right), \tag{46}$$

where the free spin-3/2 Lagrangian is now given by Eq. (2), while the rest of the terms in $\mathcal{L}$ remain unchanged. Note that starting from the transverse gauge we would obtain the same expression.

Let us now reconstruct the Feynman rules for the RS field. The delta functions in our final path integral clearly indicate that the Green functions are independent of the spin-1/2 sector of ψ_{μ}. We can use for instance the following ''Feynman gauge'' expression for the spin-3/2 propagator:

$$S_{\alpha\beta}(p)=\frac{1}{\not p-m}(g_{\alpha\beta}-\tfrac{1}{3}\gamma_\alpha\gamma_\beta). \tag{47}$$

The expression for the vertex reads

$$\Gamma^\mu(k,p)=ig\varepsilon^{\mu\nu\alpha\beta}p_\nu\gamma_5\gamma_\alpha k_\beta, \tag{48}$$

where k is pion momentum, while p can be chosen to represent the momentum of either the Δ or the nucleon.

Using these rules we can easily compute the tree-level amplitude for the πN scattering through the s- or u-channel Δ exchange (forgetting about the isospin)

$$M(k',k;p)=\Gamma^\alpha(k',p)\,S_{\alpha\beta}(p)\,\Gamma^\beta(k,p)$$

$$=\frac{g^2}{\not p-m}p^2\,P^{3/2}_{\alpha\beta}(p)\,k'^\alpha k^\beta, \tag{49}$$

where

$$P^{3/2}_{\alpha\beta}(p)=g_{\alpha\beta}-\tfrac{1}{3}\gamma_\alpha\gamma_\beta-\frac{1}{3p^2}(\not p\gamma_\alpha p_\beta+p_\alpha\gamma_\beta\not p), \tag{50}$$

is the spin-3/2 projection operator. This operator has the well-known property of projecting on the spin-3/2 states and is a clear signature of the spin-3/2 components. Our amplitude is thus independent of the spin-1/2 sector of the RS field, which is certainly the result we desired to obtain.

The spin-3/2 projection operator was used previously in some phenomenological models although in a rather *ad hoc* way, such as, for example, replacing the tensor part of the RS propagator by the projection operator, etc., see, e.g., references cited in [4]. However, in these models problems arise due to the $1/p^2$ nonlocality of the projection operator. In Eq. (49) this problem is obviously not present, which is not surprising since we depart from a local Lagrangian.

It may look that the JS-VZ problem for coupling (40) is avoided just because we made use of the Stückelberg mechanism: θ_4 is then guaranteed to be the first-class constraint and the problem discussed below Eq. (28) cannot occur. Suppose, however, we do not introduce the Stückelberg field. In this case, θ_4 is given by Eq. (41) with $\xi=0$, and the commutator of θ_4 constraints is given by Eq. (13), i.e., is exactly the same as in the free theory. Thus, the JS-VZ problem does not occur here, independently of whether the Stückelberg field is used or not.

On the other hand, suppose we would like to avoid the JS-VZ problem in the conventional coupling by using the Stückelberg mechanism. Then, indeed, the corresponding θ_4 constraint becomes first class, hence its commutator vanishes instead of being field-dependent as in Eq. (28). In that case, however, the Stückelberg field does not ever decouple and the excess of DOF becomes thus explicit, leading again to the unitarity problem.

V. SUMMARY AND CONCLUSION

The Dirac-Faddeev quantization method is very well suited for analyzing the interacting spin-3/2 field, since it provides a straightforward procedure where the control over the degrees of freedom can be done in a simple transparent way. We have applied this procedure to the conventional $\pi N\Delta$ coupling, Eq. (1), and find this coupling has a number of problems precisely due to the coupling to extra DOF. This goes in line with some previous analyses [2,16,17], as well as with the common knowledge that this coupling always produces unphysical spin-1/2 backgrounds in addition to the spin-3/2 contribution. For the choice $a=-1$, the problem is not so pronounced, nevertheless it is present and can be related to the well-known JS-VZ problem. Furthermore, we argue that for this choice the ''naive'' Feynman rules may be unapplicable since in principle there are contributions from the determinant in the path integral Eq. (30).

Further, we have suggested to use couplings which are invariant under the gauge transformation of the RS field Eq. (14). As has been conjectured and partially proved in Sec. IV, these couplings are generally consistent with the DOF counting (unitarity). We have considered two lowest in derivatives gauge-invariant $\pi N\Delta$ couplings. The first one describes the coupling to purely the spin-1/2 sector of the RS field, and we abandon its further analysis for this reason. The second coupling, Eq. (40), describes the coupling to purely spin-3/2 sector of the RS field. This conclusion is derived both nonperturbatively from the resulting path integral (46), and perturbatively from the calculation of the tree-level amplitude, Eq. (49). The gauge-invariant coupling Eq. (40) is thus a good candidate for a consistent cubic interaction of a scalar, spinor and vector-spinor fields in flat Minkowski space time.

Some other consistent interactions of the spin-3/2 field can be immediately written down knowing that they should be restricted by gauge invariance. For instance,

$$\mathcal{L}_{\pi\Delta\Delta}=g_{\pi\Delta\Delta}\overline{\psi}_\mu\gamma_5\tilde{G}^{\mu\nu}\partial_\nu\phi, \tag{51}$$

$$\mathcal{L}_{\gamma N\Delta}=g_{\gamma N\Delta}\overline{\Psi}\Theta_{\alpha\beta,\mu\nu}G^{\alpha\beta}F^{\mu\nu}+\text{H.c.}, \tag{52}$$

where $F^{\mu\nu}$ is the electromagnetic field strength, Θ is a constant tensor, e.g.,

$$\Theta_{\alpha\beta,\mu\nu}=g_{\alpha\mu}g_{\beta\nu}+a_1 g_{\alpha\mu}\gamma_\beta\gamma_\nu+a_2\varepsilon_{\mu\nu\alpha\beta}$$
$$+\text{derivative terms}, \tag{53}$$

and, finally, $G_{\mu\nu}=\partial_\mu\psi_\nu-\partial_\nu\psi_\mu$, $\tilde{G}^{\mu\nu}=\varepsilon^{\mu\nu\alpha\beta}\partial_\alpha\psi_\beta$.

An acceptable $\gamma\Delta\Delta$ interaction can also be easily found as long as the coupling to the photon is ''anomalous,'' i.e., occurs only through $F^{\mu\nu}$. On the other hand, to write down a consistent minimal coupling is not a trivial task since it is then difficult to satisfy both photon and spin-3/2 gauge symmetries at the same time. In this case, as well as in other cases when one needs to set up lower-derivative interactions, supersymmetry might be the only option.

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APPENDIX: STÜCKELBERG MECHANISM FOR THE SPIN-3/2 FIELD

Our procedure goes in full analogy to the massive spin-1 case (the Proca model). We introduce a ''Stückelberg spinor'' $\xi(x)$ replacing ψ_μ by $\psi_\mu+\partial_\mu\xi$ in the free Lagrangian (2). The Lagrangian reads then as follows:

$$\mathcal{L}_{3/2}=\tfrac{1}{2}\overline{\psi}_\mu\{\sigma^{\mu\nu},(i\partial\!\!\!/-m)\}\psi_\nu - m(\partial_\mu\overline{\xi})\sigma^{\mu\nu}\psi_\nu - m\overline{\psi}_\mu\sigma^{\mu\nu}\partial_\mu\xi, \tag{A1}$$

and it is manifestly invariant under the gauge transformation

$$\psi_\mu\to\psi_\mu+\partial_\mu\epsilon, \qquad \xi\to\xi-\epsilon. \tag{A2}$$

We define the conjugate momenta[8]

$$\pi^{\mu\dagger}(x)=\partial\mathcal{L}/\partial\dot{\psi}_\mu(x), \qquad \eta^\dagger(x)=\partial\mathcal{L}/\partial\dot{\xi}(x), \tag{A3}$$

and fundamental Poisson brackets

$$\{\psi_{\mu\tau}(x),\,\pi^{\nu\dagger}_\sigma(y)\}_P=\delta^\nu_\mu\delta_{\sigma\tau}\delta^3(x-y), \qquad \{\xi_\tau(x),\,\eta^\dagger_\sigma(y)\}_P=\delta_{\sigma\tau}\delta^3(x-y), \tag{A4}$$

where $\tau,\sigma=0,\ldots,3$ are the spinor indices. We obtain then the following primary constraints:

$$\theta_0(x)=\pi_0(x), \qquad \theta_i(x)=\pi_i(x)-\frac{i}{2}\sigma_{ij}\psi_j(x), \qquad \theta_S(x)=\eta(x)-m\gamma_i\psi_i(x), \tag{A5}$$

and the Hamiltonian

$$H_{3/2}=\int d^3x\,\mathcal{H}_{3/2}, \qquad \mathcal{H}_{3/2}=\psi_0^\dagger\theta_4+\tfrac{1}{2}\overline{\psi}_i(\varepsilon_{ijk}\gamma_0\gamma_5\partial_k+m\sigma_{ij})\psi_j + m\overline{\psi}_i\sigma_{ij}\partial_j\xi+\text{H.c.}, \tag{A6}$$

where θ_4 is the only secondary constraint, given by

$$\theta_4(x)=-i\sigma_{ij}\partial_i\psi_j(x)+m\gamma_i[\psi_i(x)+\partial_i\xi(x)]. \tag{A7}$$

We introduce the Dirac bracket with respect to the second-class constraints θ_i and θ_S. Using this bracket the field commutators take the following form:

$$\{\psi_i(x),\psi_j^\dagger(y)\}_D=-i(\delta_{ij}+\tfrac{1}{3}\gamma_i\gamma_j)\delta^3(x-y), \qquad \{\xi(x),\xi^\dagger(y)\}_D=-i\frac{2}{3m^2}\delta^3(x-y), \qquad \{\xi(x),\psi_i^\dagger(y)\}_D=\{\psi_i(x),\xi^\dagger(y)\}_D=\frac{1}{3m}\gamma_i\delta^3(x-y). \tag{A8}$$

We find then that the secondary constraint commutes with the Hamiltonian, i.e.,

$$\{\theta_4,H_{3/2}\}_D=0, \tag{A9}$$

thus no further constraints arise. We also conclude that θ_0 and θ_4 are the first-class constraints.

Let us denote the first-class constraints as $\theta^{(1)}=(\theta_0,\theta_4)$, the corresponding gauge-fixing conditions as $\varphi=(\varphi_1,\varphi_2)$, and the second-class constraints as $\theta^{(2)}=(\theta_i,\theta_S)$. Then, assuming φ's commute among themselves, the path integral can be put in the following form (see, e.g., [22,23]):

$$Z=\int\mathcal{D}\psi_\mu\,\mathcal{D}\psi_\mu^\dagger\,\mathcal{D}\xi\,\mathcal{D}\xi^\dagger\,\mathcal{D}\pi^\mu\,\mathcal{D}\pi^{\mu\dagger}\mathcal{D}\eta\,\mathcal{D}\eta^\dagger \times\det\|\{\theta^{(1)},\varphi\}_D\|\,(\det\|\{\theta^{(2)},\theta^{(2)}\}_P\|)^{1/2} \times\prod\,\delta(\varphi)\,\delta(\varphi^\dagger)\,\delta(\theta)\,\delta(\theta^\dagger) \times\exp\left\{i\int d^4x[\pi^{\mu\dagger}\dot{\psi}_\mu+\dot{\psi}_\mu^\dagger\pi^\mu+\eta^\dagger\dot{\xi}+\dot{\xi}^\dagger\eta-\mathcal{H}_{3/2}]\right\}. \tag{A10}$$

In our case $\det\|\{\theta^{(2)},\theta^{(2)}\}_P\|$ is just a constant and can be dropped, since the path integral is defined up to a normalization factor.

Clearly one of the gauge-fixing conditions must be proportional to ψ_0 in order to match the θ_0 constraint. We take $\varphi_1=\psi_0$, then for φ_2 there is a number of possibilities, e.g.,

$\varphi_2=\gamma_i\psi_i$ (Coulomb gauge),

$\varphi_2=\partial_i\psi_i$ (transverse gauge),

$\varphi_2=\psi_3$ (axial gauge),

$\varphi_2=\xi$ (unitary gauge).

Let us choose the Coulomb gauge. Integrating over the conjugate momenta we then arrive at

[8]We shall omit similar formulas for the Hermitian-conjugate fields where possible.

$$Z=\int \mathcal{D}\psi_\mu \mathcal{D}\psi_\mu^\dagger \mathcal{D}\xi \mathcal{D}\xi^\dagger (\det[\gamma_i\partial_i])^2 \times \delta(\gamma_i\psi_i)\delta(\psi_i^\dagger\gamma_i)e^{i\int \mathcal{L}_{3/2}}. \quad \text{(A11)}$$

Now, having the gauge symmetry at our disposal, we may use the Faddeev-Popov trick [19,24,35] to covarianize the measure. We thus obtain the path integral in a covariant gauge,

$$Z=\int \mathcal{D}\psi_\mu \mathcal{D}\psi_\mu^\dagger \mathcal{D}\xi \mathcal{D}\xi^\dagger (\det[\not\partial])^2 \times \delta(\gamma\cdot\psi)\delta(\psi^\dagger\cdot\gamma)\exp(i\int \mathcal{L}_{3/2}). \quad \text{(A12)}$$

Starting from the transverse gauge we would arrive at

$$Z=\int \mathcal{D}\psi_\mu \mathcal{D}\psi_\mu^\dagger \mathcal{D}\xi \mathcal{D}\xi^\dagger (\det[\partial_\mu\partial^\mu])^2 \times \delta(\partial\cdot\psi)\delta(\partial\cdot\psi^\dagger)\exp\left(i\int \mathcal{L}_{3/2}\right). \quad \text{(A13)}$$

In these gauges the massless limit can be obtained directly. On the other hand, taking the unitary gauge and integrating over ξ's gives us back Eq. (18).

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